



# FIVE AND NINE PLUS

THE OFFICIAL NEWSLETTER  
OF THE  
APPLEDORE AND DISTRICT  
AMATEUR RADIO CLUB

Club Callsigns: G2FKO and GX2FKO  
Web Site : [www.adarc.co.uk](http://www.adarc.co.uk)

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January 2021

## EDITORIAL

Welcome to the first Newsletter of 2021 - where do the years go?!!

Our first Meeting this year is next **Monday, January 18th** starting at 7.30pm and will be a **Zoom talk by Mike (G4KXQ)** who has offered to give us what will be a most interesting talk on the new **Icom IC705 rig** which Mike was lucky enough to have from Father Xmas! So not too much time for Mike to prepare but we are extremely grateful to him for his offer to give us this talk - I'm sure the bruises will have healed by next Monday!! Mike's talks are always very well prepared and most professionally presented so it promises to be yet another fascinating talk. I must say that as a Club with a modest membership we are extremely fortunate in the enthusiasm of our members.

So it only remains for me on behalf of your Committee to wish everyone a Happy Healthy New Year.

Keep safe and keep healthy

Terry (G4CHD)



## CLUB MEETING

Due to the present Covid 19 pandemic, **ALL meetings scheduled to be held at the Appledore Football Social Club have been CANCELLED until further notice**

Until Meetings are reinstated a programme of 'virtual meetings using Zoom' has been arranged :-

Date	Topic
Jan 18 <sup>th</sup>	Zoom Talk - "IC 705" by Mike (G4KXQ)

It is hoped to add further Zoom Meetings so if you want to give a talk, please contact any Committee member.

## DECEMBER ZOOM MEETING 'CHRISTMAS PARTY'

A small number of members met on December 21<sup>st</sup> and enjoyed a pleasant chat and the odd mince pie etc. Dave (G0PGK) had kindly spent time preparing a Christmas Quiz (complete with festive jumper etc!) but with such a poor turnout, it was decided to abort the quiz which was a great shame after all Dave's work.

However those taking part including myself had a most enjoyable time and got us into the Christmas spirit.

The lack of participating members does raise the question as to what the Club should be doing in these difficult times when our face to face Meetings which we all long for are out of the question.

## LOCAL REPEATERS

**2m Stibb Cross Repeater (GB3DN)**

<http://www.g0rql.co.uk/gb3dn.htm>

User: Listen 145.6375 MHz - Transmit 145.0375 MHz.

Access 1750 Hz Tone or 77 Hz CTCSS

Repeater keeper is Tony (G1BHM)

**Fusion/C4FM/WiresX Gateway (MB6DT)**

Frequency 144.8125 MHz.

Keeper Darren (2E0LVC)

**Fusion/C4FM/WiresX Gateway (MB6DN)**

Frequency 144.825 MHz.

Keeper Drew (2E0FQE)

**LOCAL NETS**

**Zepp FM Net:** Mon/Tues/Thurs/Fri :  
145.450MHz - 4pm - 5pm  
Wed via GB3DN - 4pm - 5pm

**2m Elevenes FM Net:** Mon/Tues/Thurs :  
11 - 12.00 noon 145.475MHz  
Wed/Fri :  
11 - 12.00 noon via GB3DN

**Friday Night 2m Net:** Friday : 145.450 FM, 8 - 9pm

**2m SSB Nets:** Wed: 8 - 9pm 144.260MHz  
USB SSB  
Sun: approx 10.30am (follows  
Top Band Net) 144.260MHz  
USB SSB (Vertical polarised)

**Top Band Net:** Sunday 1.860 Mhz  
9.30 - 10.15am  
(LSB - 32W pep max)

**CHANGE TO THE 11'S NET**

As of Friday January 15<sup>th</sup> the 11's Net run by Mike (G3PGA) will be on the GB3DN Repeater every Wednesday AND Friday to make it easier for distant amateurs to join in.

**CROSSWORD**

Many thanks to Stuart (M1FWD) for this month's Crossword.  
The answers are in next month's Newsletter. Good luck !

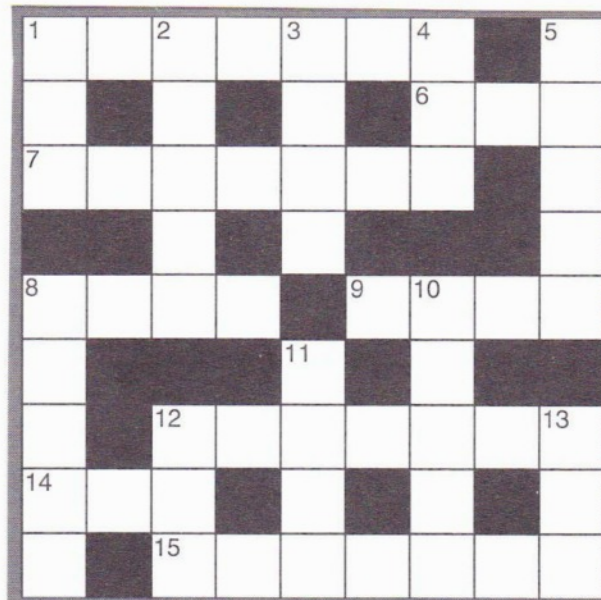


**CLUES ACROSS**

- 1) Base for amateur radio operations (7)
- 6) A unit of energy or work in the centimetre-gram-second system of physical units used in physics (3)
- 7) Charlie Tango Three (CT3) island (7)
- 8) A magnetic structural unit in a computer, storing one bit of data (4)
- 9) Capital city of Oscar Alpha (OA) land (4)
- 12) ? circuit, an electric circuit with thin strips of conductor on a flat insulating sheet (7)
- 14) Drink made from the leaves of the camellia plant (3)
- 15) Inflammation of the mucous membrane of the nose (7)

**CLUES DOWN**

- 1) The total amount (3)
- 2) The common viper (5)
- 3) Flat circular coloured membrane behind the cornea of the eye (4)
- 4) Popular female forename in Oscar Hotel (OH) land (3)
- 5) Market place and forum in ancient Sierra Victor (SV) land (5)
- 8) Sierra Victor Nine (SV9) island (5)
- 10) Bury (5)
- 11) One-eighth of a gallon (4)
- 12) ? Man, video game series (3)
- 13) The 1st and 8th notes of a major musical scale (3)



Last month's answers :-

**ANSWERS ACROSS:** 1) loss 5) Cruel 7) neutron  
8) The City 11) Reeboks 13) tuner 14) Asia

**ANSWERS DOWN:** 1) long 2) Southern 3) Pro 4) plug  
5) cracker 6) unctuo

**SUDOKU PUZZLE**

The aim is to enter a number into each cell so that any column, or any row, or any block of cells contains all numbers from 1 to 9  
Terry (G4CHD)

					2	4		1
				9	7		2	
			4	8		7	6	9
								8
4		3				9		6
		9						
3	1	4	2	6				
	9		3		8			
6		2			4			

**ITEM FOR SALE**

Mike (G3PGA) is having a mini shack clearance and so has for sale :-

**Yaesu VX-170 handie** in original box with manual, charger as new. Price £80

If interested please contact Mike at :-  
[g3pgamike@gmail.com](mailto:g3pgamike@gmail.com)

## TECHNICAL ARTICLE

Finally, being at the usual loss as to what technical content to include and being spoilt for choice, I have included an article I recently wrote (mainly for my own attempt to understand it !) on the **Smith Chart**. The intention is to follow it up with another article which would try to examine how using the Smith Chart needs to be modified with lossy cables (the Smith Chart assumes loss free cables) - so be warned! I must add that reading the article comes with a boredom warning!!

The article follows on the following pages.

So that's it for this month

Hopefully not too many errors this month!!

Terry (G4CHD)

## SMITHS CHART DERIVATION AND USE - AN ARTICLE BY TERRY G4CHD

We will first look at the way in which a transmission line works :-

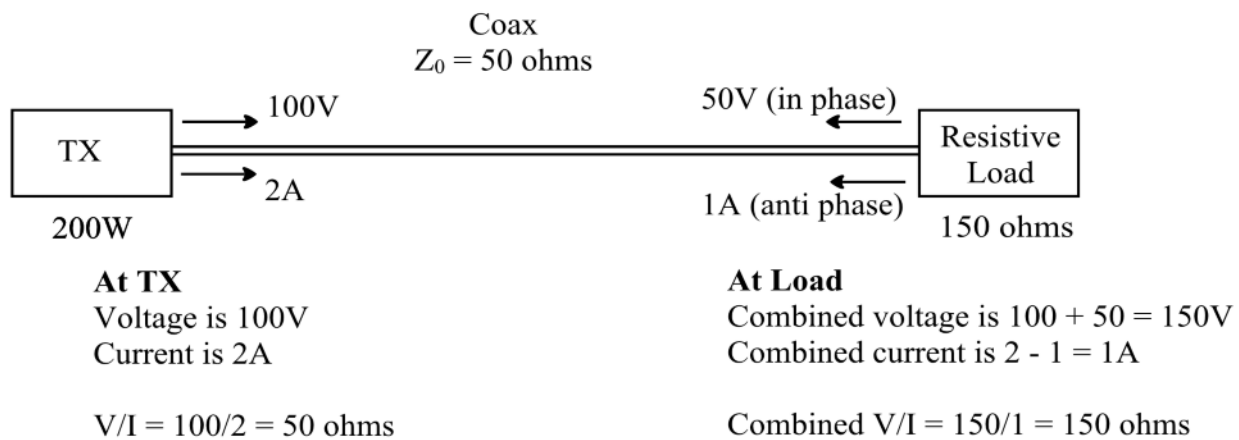
Consider a 200 Watt transmitter feeding a transmission line of 50 Ohms (resistive) Characteristic Impedance ( $Z_0$ ). The transmitter will treat the transmission line as a 50 Ohm resistive load, and assuming a transmitter voltage of 100 Volts RMS then in order for Ohms Law to be satisfied, it will deliver a current of 2A RMS into the transmission line. The transmitter is therefore delivering  $100 \times 2 = 200$  Watts power into the transmission line. The voltage and current waves travel as incident waves along the line until they reach the load impedance at the far end which will be assumed initially to be a pure resistance eg a resonant antenna.

If the resistive load impedance is exactly equal to and thus matched to the 50 ohms transmission line, then the 100 volt and 2 amp waves arriving at the load satisfy Ohms Law and all 200 Watts (assuming no line losses) will be delivered to the load.

However this is a very idealistic situation as even if the load is purely resistive, it is more likely that its resistive impedance is not equal to 50 Ohms.

### Consider a load resistance of 150 Ohms :-

The problem of satisfying Ohms Law is solved by reflected Voltage and Current waves being generated which travel back down the transmission line towards the transmitter. At the 150 ohms load, the combination of incident and reflected voltage and current waves now satisfy Ohms Law. As the resistive load is greater than the 50 ohms  $Z_0$  of the line, the reflected Voltage wave is in phase and thus additive with the incident wave, whereas the current reflected wave is in anti phase and thus subtractive with the incident wave as summarised in the following diagram :-



As the reflected voltage and current waves travel towards the transmitter, at a distance of a quarter of an electrical wavelength from the load, the reflected voltage and incident waves are now in anti phase and the reflected and incident current waves are in phase. Thus at a distance of a quarter of an electrical wavelength from the load, the combined voltages are  $100 - 50 = 50$ V and combined currents are  $2 + 1 = 3$ A which according to Ohms Law, gives a resistive impedance of  $50/3 = 16.66$  ohms. After travelling a further quarter of an electrical wavelength towards the transmitter, the phases revert back to the values at the load and the impedance is thus the same as that of the resistive load. It must be emphasised that this is **ONLY** true for a transmission line have **NO** losses.

The magnitude of the fraction of incident wave reflected is termed the Reflection Coefficient **k** (later we shall see that the Reflection Coefficient is actually a complex number  $\Gamma$ ).

As seen above, the combined voltage varies along the line with a maximum value of 150V (sum of incident and reflected waves) and a minimum of 50V (difference of the incident and reflected waves). The ratio of these

extremes of voltage is termed the (Voltage) Standing Wave Ratio - (V)SWR or just SWR. It is convenient to derive formulae relating these various parameters as shown below :-

Let Incident Voltage and Current be  $V_i$  and  $I_i$  Let

Reflected Voltage and Current be  $V_r$  and  $I_r$  Let

Reflection Coefficient (Ratio) be  $k$

Assume transmission line is **LOSS FREE**

Transmission Line Characteristic Impedance  $Z_0$  is 50 ohms (resistive)

Assume Load Impedance  $Z_L$  is purely resistive and greater than  $Z_0$  Then

∴  $V_i/I_i = Z_0$

$k = V_r/V_i$  and also  $k = I_r/I_i$

Since  $Z_L = (V_i + V_r)/(I_i - I_r) = V_i(1 + V_r/V_i)/I_i(1 - I_r/I_i) = V_i(1 + k)/I_i(1 - k) = Z_0(1 + k)/(1 - k)$

then  $Z_L(1 - k) = Z_0(1 + k)$  from which we get  $k = (Z_L - Z_0)/(Z_L + Z_0)$  .....equation 1

However, if  $Z_L < Z_0$  then  $k = (Z_0 - Z_L)/(Z_0 + Z_L)$

$SWR = (V_i + V_r)/(V_i - V_r) = (1 + V_r/V_i)/(1 - V_r/V_i) = (1 + k)/(1 - k) = SWR$  equation 2

from which we get  $k = (SWR - 1)/(SWR + 1)$

Combining equations 1 & 2 we get  $SWR = Z_L/Z_0$  equation 3

However, if  $Z_L < Z_0$  then  $SWR = Z_0/Z_L$

Finally it can be shown that :-  $Z_{max} = Z_0(1 + k)/(1 - k)$  equation 4

$Z_{min} = Z_0(1 - k)/(1 + k)$  equation 5

Using the above formulae and assuming  $V_i = 100V$  and  $I_i = 2A$  - we thus get for various resistive loads :-

$Z_L$	$k$ (1)		$V_i$	$V_r$	$V_{max}$	$V_{min}$	$Z_{max}$ (4)	$Z_{min}$ (5)
5	0.818	10	100	81.8	181.8	18.2	500	5
10	0.666	5	100	66.6	166.6	33.4	250	10
20	0.429	2.5	100	42.9	142.9	57.1	125	20
30	0.25	1.6	100	25	125	75	80	31.2
40	0.166	1.25	100	16.6	116.6	83.4	62.5	40
50	0	1	100	0	100	100	50	50
100	0.333	2	100	33.3	133.3	66.7	100	25
150	0.5	3	100	50	150	50	150	16.7
200	0.6	4	100	60	160	40	200	12.5
300	0.714	6	100	71.4	171.4	28.6	300	8.3
400	0.777	8	100	77.8	177.8	22.2	400	6.2
500	0.818	10	100	81.8	181.8	18.2	500	5
1000	0.905	20	100	90.5	190.5	9.5	1000	2.5

It can thus be seen that as the SWR increases, the extremes of voltage (and current) worsen leading to losses in the transmission line, and extremes in line impedance make matching the to the transmitter more difficult.

In the previous analysis, it was assumed that the transmission line fed a purely resistive load - eg a resonant antenna at the operating frequency. However, this is an ideal situation which is often difficult to achieve in practice. More usually the load is a complex impedance  $Z$  having both resistance  $R$  and reactance  $X$  (eg a non resonant antenna).

## COMPLEX IMPEDANCE

The resistance  $R$  of an antenna measured in ohms can be considered as a way of indicating that it is dissipating (radiating) radio energy and thus absorbing power from the transmitter.

Since Power =  $I^2 \times R$  then in the previous example where the antenna was matched to the transmission line such that all the 200W power from the transmitter (line considered to be loss free) is absorbed by the antenna, then since the incident current was 2A then it follows that using Power =  $I^2 \times R$  then  $R = 200/2 \times 2 = 50$  ohms.

There will in addition be a very small amount of heat energy dissipated in the antenna due to the resistance of the material from which is made. Since this will usually be 1 ohm or less then less than 4W will be dissipated in the antenna as heat. It follows that for antennas having a high resistive impedance and therefore drawing less current, such losses will be reduced.

In fact resistance is the **ONLY** type of impedance that can absorb/dissipate energy.

Finally, the voltage and current waveforms relating to resistance are **IN PHASE** with each other.

Reactance on the other hand **CANNOT** absorb/dissipate energy so never buy a heater having no resistance!!

There are two types of reactance - inductive  $X_L$  and capacitive  $X_C$ .

Inductive reactance  $X_L$  is derived from a coil's inductance  $L$  measured in Henries where  $X_L = 2\pi fL$  and  $X_C$  is derived from a capacitor's capacitance measured in Farads where  $X_C = 1/2\pi fC$ . As can be seen, reactance is frequency dependent.

For both inductive and capacitive reactance, current is 90 degrees out of phase with the voltage. There is a way to remember whether current leads or lags the voltage using the word **CIVIL**

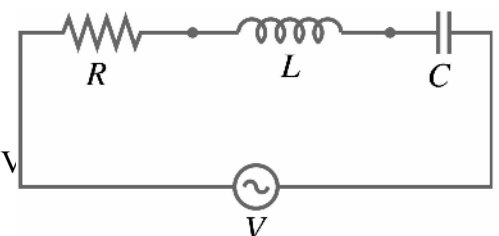
ie for a capacitor with capacitance **C** - current leads voltage  $V$  whereas for an inductor with inductance **L** - current  $I$  lags voltage  $V$ .

Consider a series circuit containing resistance  $R$ , inductance  $L$  and capacitance  $C$

The total impedance  $Z$  is the sum of resistance  $R$ , inductive reactance  $X_L$  and capacitive reactance  $X_C$ . However a way is needed to indicate the phase differences between resistance and both types of reactance.

This is done mathematically using the symbol  $j$  which when positive indicates  $V$  leads  $I$  (as for an inductor) and when negative indicates that  $V$  lags  $I$  (as for a capacitor). Consequently for the circuit shown we can write :-

$$Z = R + jX_L - jX_C$$



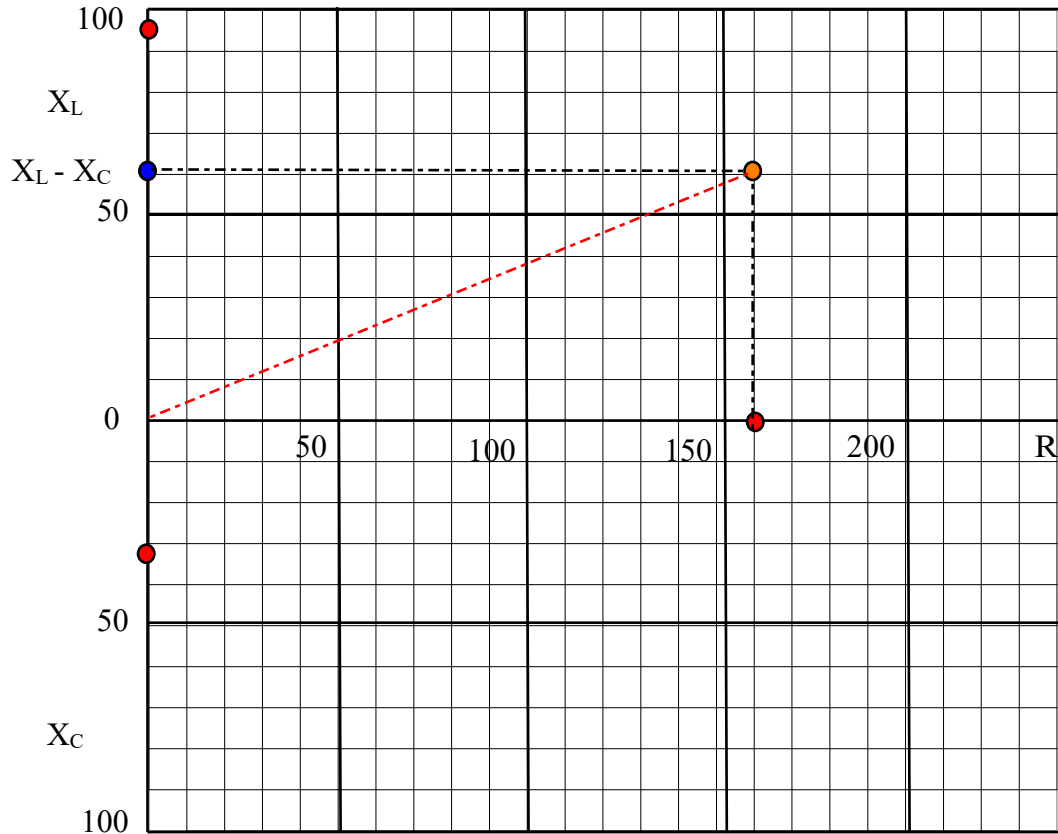
It is also convenient to plot resistance and reactance on a Vector Impedance diagram where the X axis plots resistance  $R$  values and the Y axis plots inductive reactance  $X_L(+)$  vertically upwards and capacitive reactance  $X_C(-)$  downwards.

Consider a series circuit consisting of 160 ohms resistance in series with 15uH inductance and 5 nF capacitance. Let the frequency of the applied voltage be 1 MHz.

15uH inductor has an inductive reactance  $X_L$  of  $2\pi fL = 2 \times 3.14 \times 1 \text{ million} \times 15 \text{ millionths} = 94 \text{ ohms}$

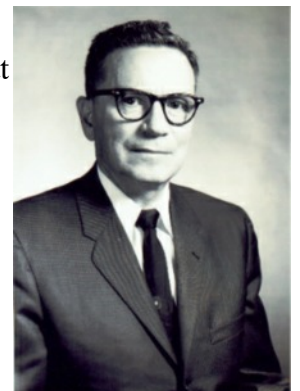
5nF capacitor has a capacitive reactance  $X_C$  of  $1/2\pi fC = 1/(2 \times \pi \times 1 \text{ million} \times 5 \text{ thousand millionths}) = 32 \text{ ohms}$

Plotting these on the Vector Impedance graph shown below:-



The two reactances can be subtracted as they have opposite phase relationships to give 62 ohms inductive reactance. Combining this with the 160 ohms resistance using the Pythagoras Rule gives us point with a value of 160 ohms resistive and 62 ohms inductive reactance. The length of the dotted red line gives the total impedance of the circuit which is 172 ohms and the angle between the red dotted line and the resistance axis gives the final phase difference between the applied voltage and the current in the circuit ie the current lags the voltage by approx 21 degs. Hence we can write  $Z = 160 + j62$ .

Now all this will have probably completely lost your interest by now but goes to show that once you are dealing with complex impedances, the mathematics become very complicated. It was for this reason to analyse transmission lines feeding complex impedance loads, a relatively simple and very clever graphical method was devised by Phillip Hagar Smith in 1939 whilst working for the Bell Telephone Labs in the States called a Smith Chart. In the next section I will show how you can produce a Smith Chart and this will be followed by a section showing how it can be used to solve a variety of antenna problems together with practical examples at my QTH.



### SMITH CHART - HOW TO DRAW ONE

Unlike an Impedance Graph as shown above which uses a linear rectangular grid, the Smith Chart uses a circular grid with a logarithmic scale. A more mathematical explanation of the Smith Chart is given in the Appendix.

The starting point is a circular disc of a diameter of your choosing. I will adopt a value of 20cm as the diameter as it just fits onto an A4 page.

Draw a vertical diametrical line which will be used to plot impedance having only resistance and no reactance with the value of  $Z_0$  (50 ohms) at its centre and draw the first circular resistance grid line for 50 ohms with a diameter of 10cm such that the circle fits between the centre point and the base of the outer circle.

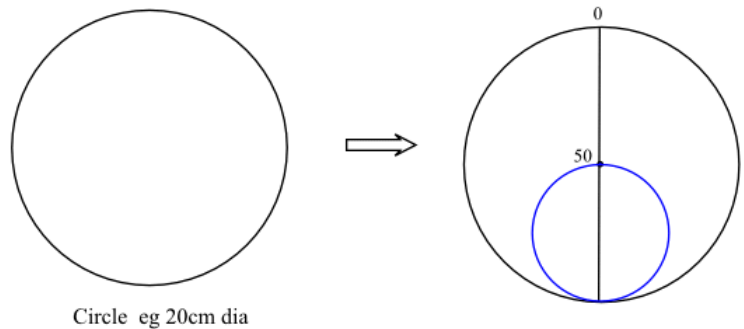
Difficult to explain but hopefully the diagram on the following page will make it much clearer :-

Circles for other pure resistance values have diameters which vary non linearly according to the following formula which assumes  $Z_0$  is 50 ohms and the outer circle has a 20cm diameter:-

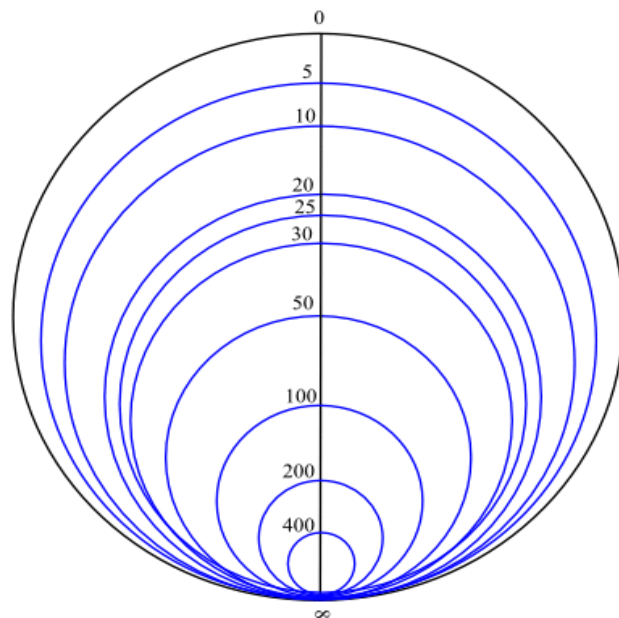
$$\text{Diameter (cm)} = 20 \times 50 / (50 + R)$$

Hence tabulated diameter values for various resistance values are :-

R ohm	Dia
cm	
5	18.2
10	16.7
20	14.3
25	13.3
30	12.5
40	11.1
50	10.0
100	6.7
150	5.0
200	4.0
300	2.9
400	2.2
500	1.8



These resistance circles are plotted below :-



One useful property of a Smith Chart which can now be used to check the validity of our new partly drawn Smith Chart is that a circle drawn centrally as shown below in red, represents values of **constant SWR**.

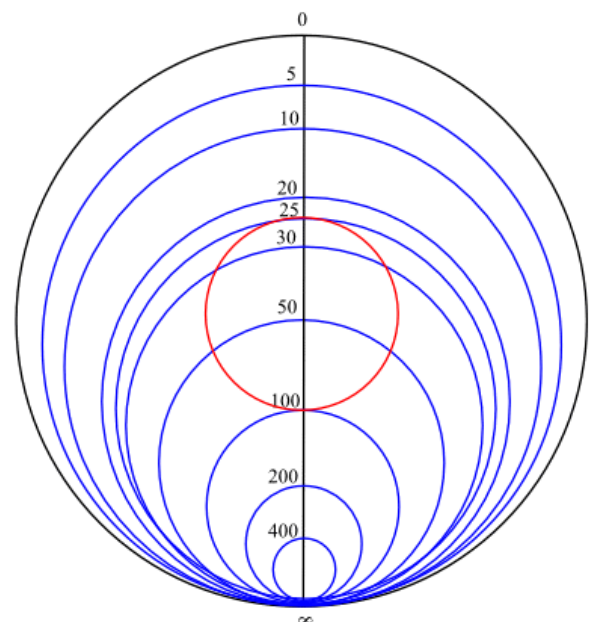
Consider a resistive load of 100 ohms then

According to equation 3 on page 2,  $SWR = 100/50 = 2$  and  $k = (100 - 50)/(100 + 50) = 50/150 = 0.33$

According to equations 4 and 5, the extreme values of resistive impedance are  $50 (1 - 0.33)/(1 + 0.33) = 25$  ohms

and  $50 (1 + 0.33)/(1 - 0.33) = 100$  ohms

Thus if the circle in red represents an  $SWR = 2$  then it should pass through both points 25 and 100 ohms which thankfully it does!



Note that the outer blue 5 ohms resistance circle has a diameter of 18.2 cm.

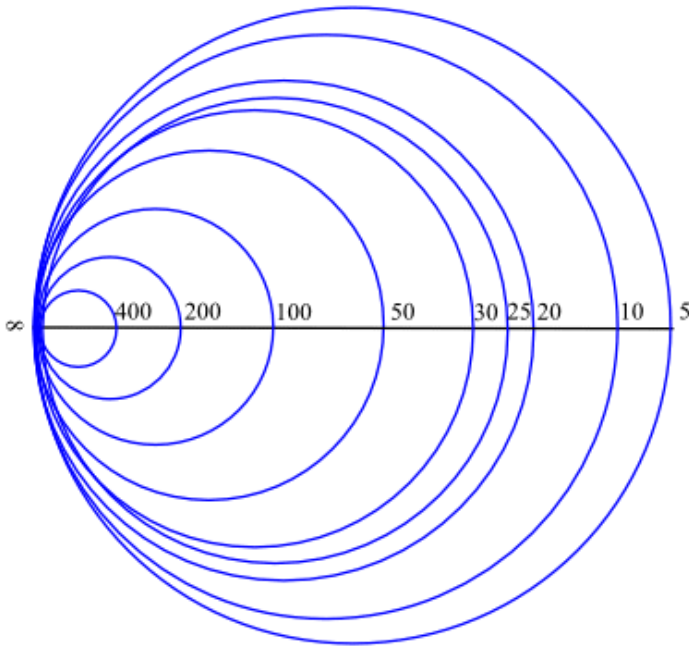


We must now turn our attention to introducing a series of circles representing various **values of both positive and negative reactance** so that complex impedances can be plotted.

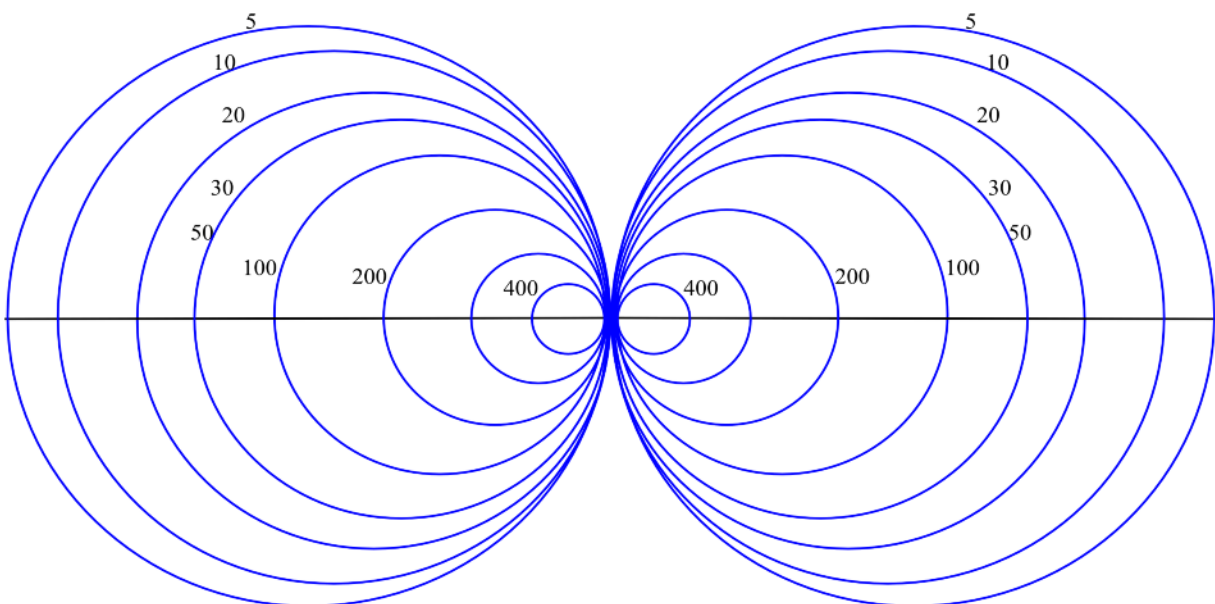
As with a normal Impedance Graph, the resistance and reactance axes are at right angles. Similarly to some extent with a Smith Chart, the circles of resistance lie on a line at right angles to the line upon which the circles of reactance are centred. Assuming as before that  $Z_0 = 50$  ohms and the outer circle has a diameter of 20 cm then the diameter of each reactance circle is given by the formula which is used to produce the tabulated values shown opposite :-

Diameter (cm) =  $40 \times Z_0 / (Z_0 + X) = 40 \times 50 / (50 + X) = 2000 / (50 + X)$   
 where X is reactance in ohms. These are plotted in the following diagram for positive reactances. Note that the diameters are double those for the resistance circles (not drawn to scale below) and thus the 5 ohm reactance circle has a diameter of 36.4 cm :-

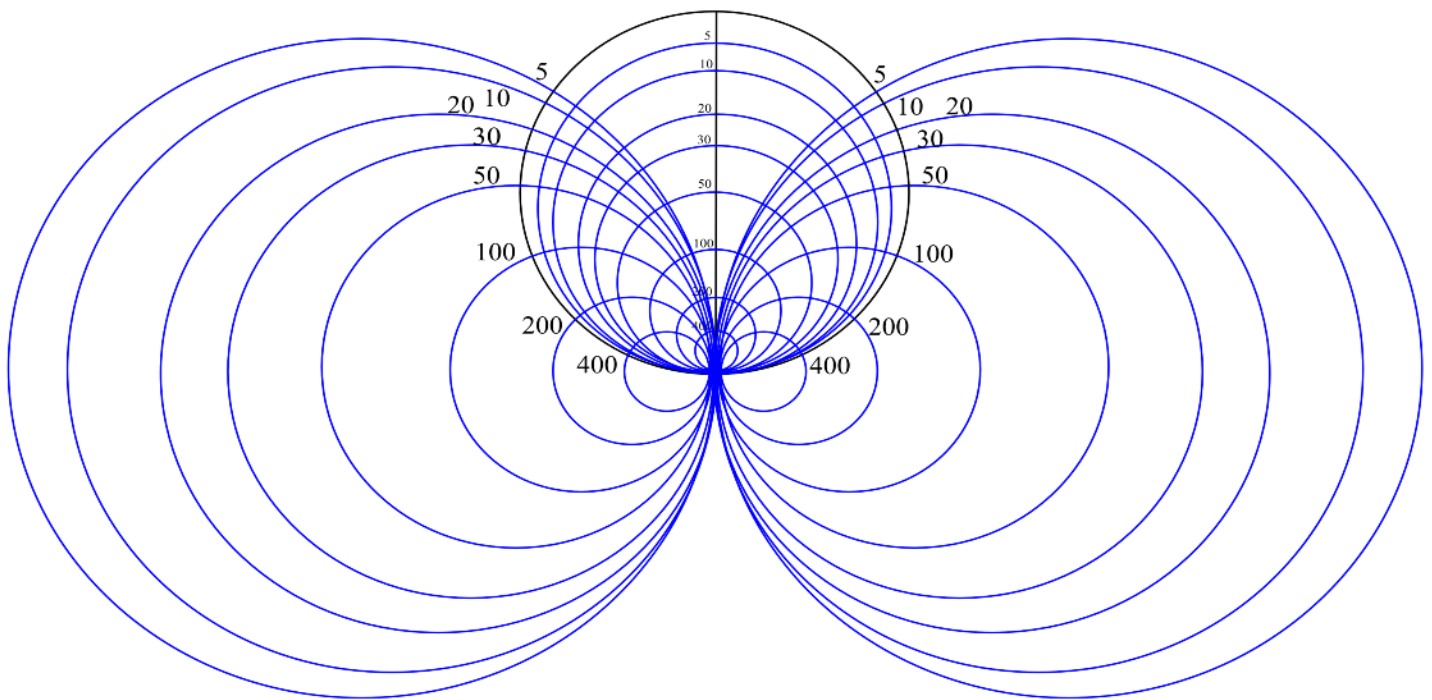
X ohm	Dia
cm	
5	36.4
10	33.3
20	28.6
25	26.7
30	25.0
40	22.2
50	20.0
100	13.3
150	10.0
200	8.0
300	5.7
400	4.4
500	3.6



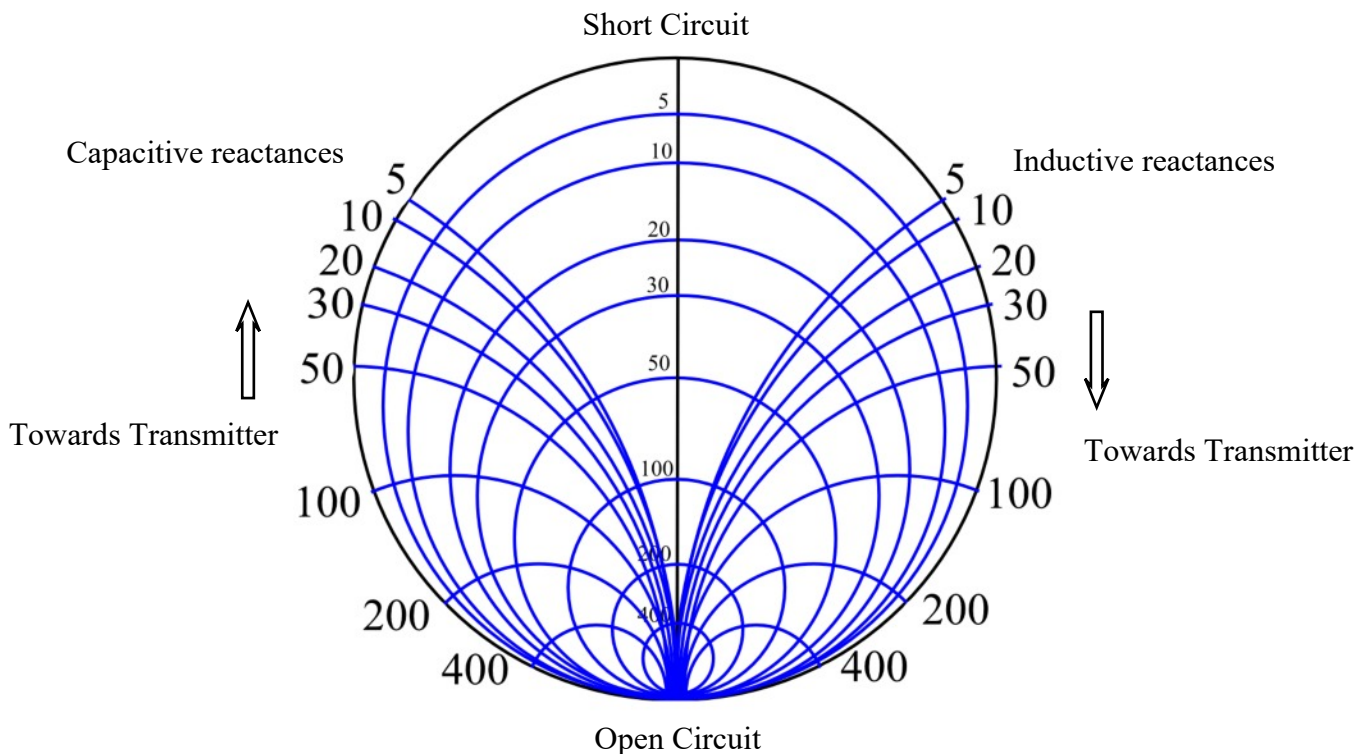
These are plotted in the following diagram for both positive and negative reactances :-



Finally, combining the resistance and reactance diagrams and allowing for the different diameters (reactance circles have twice the diameter of same value resistance circles) gives the first version of the Smith Chart as shown below.



Of course the only part of the diagram required is the region within the outer black circle :-



At the top of the outer circle we have a load of 0 ohms pure resistive ie a short circuit. Travelling clockwise along the transmission line towards the Transmitter/Generator, are values of positive inductive reactances. After 180 degs (electrical quarter of a wavelength distance from the load) at the bottom of the outer circle we get to another position of pure resistance which represents infinite value equivalent to an open circuit. Continuing clockwise along the transmission line towards the transmitter we have values of negative capacitive reactance until after 180 degs we complete the circle and the cycle is repeated.

It is therefore useful to have an outer scale to the outer circle marked in degrees or fractions of an electrical wavelength.

Finally, everything so far has assumed a transmission line Characteristic Impedance,  $Z_0$  of 50 ohms. Since there are cables with  $Z_0$  values of eg 75 ohms, or 300 ohms, Smith Charts are often 'normalised' ie all impedances are expressed as eg  $Z/Z_0$  eg a 100 ohm resistive load connected to a 50 ohms cable now has a value of 2.

## APPENDIX

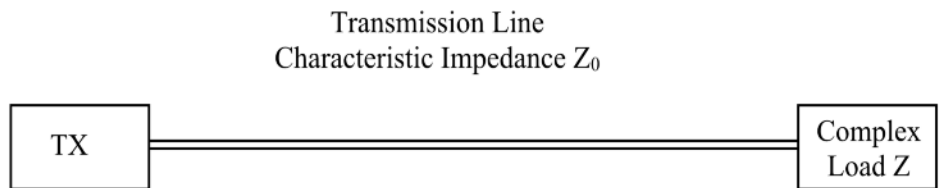
My initial thoughts on producing a Smith Chart was to treat it as a special graph using a circular grid and a non linear scale as per the main body of this article.

However, subsequently I have realised that the circular lines are in fact plotted on a linear graph as will be explained below :-

Consider a transmission line with a Characteristic Impedance  $Z_0$  feeding a complex impedance load  $Z$  (eg a non resonant antenna) :-

It is convenient to normalise all impedances such that their normalised values become  $Z/Z_0 = z$

The incident voltage and current waves from the transmitter satisfy Ohms Law  $V_i/I_i = Z_0$



Since  $Z$  is a complex impedance then  $z$  will be complex also and expressed as  $z = r + jx$  where  $r$  is real and resistive and  $x$  imaginary and reactive.

Since the load is complex and contains reactance, a fraction termed the **Reflection Coefficient  $\Gamma$**  of the incident voltage  $V_i$  and current  $I_i$  waves ( $V_r$  and  $I_r$ ) will be reflected such that at the load, the sum of the incident and reflected voltage and current waves satisfies Ohms Law ie  $(V_i + V_r)/(I_i - I_r) = Z$  and  $\Gamma = V_r/V_i = I_r/I_i$

$$\text{ie } Z = V_i(1 + V_r/V_i)/I_i(1 - I_r/I_i) = Z_0(1 + \Gamma)/(1 - \Gamma)$$

$$\text{Hence } Z/Z_0 = z = (1 + \Gamma) / (1 - \Gamma)$$

Since  $SWR = (V_i + V_r)/(V_i - V_r) = V_i(1 + V_r/V_i)/V_i(1 - V_r/V_i)$  Then  $SWR = (1 + \Gamma)/(1 - \Gamma)$

Since  $z$  is a complex impedance then it follows that  $\Gamma$  will also be complex. Hence  $\Gamma = u + jv$  where  $u$  is the real part and  $v$  the imaginary part.

We previously stated that  $z = (1 + \Gamma)/(1 - \Gamma)$  and substituting for  $\Gamma$  we get :-  $z = (1 + (u + jv))/(1 - (u + jv))$  ie

$$z = [(1 + u) + jv]/[(1 - u) - jv] \text{ which rearranged is :- } z = [1 - u^2 - v^2 + j(2v)]/[(1 - u)^2 + v^2]$$

Since  $z = r + jx$  then equating the real and imaginary parts we get :-

$$r = [1 - u^2 - v^2]/[(1 - u)^2 + v^2] \quad \text{eqn 1}$$

$$x = 2v/[(1 - u)^2 + v^2] \quad \text{eqn 2}$$

Considering **eqn 1** we know that for a fixed value of  $r$  we should get a circle on a Smith Chart.

The general eqn for a circle is of the form :-

$$(x - a)^2 + (y - b)^2 = c^2 \text{ where } a \text{ \& } b \text{ are coordinates of the centre of the circle and } c \text{ is its radius}$$

Returning therefore to **eqn 1** and rearranging it into that for a circle :-

$$r = [1 - u^2 - v^2]/[(1 - u)^2 + v^2]$$

Hence

$$r[(1 - u)^2 + v^2] = 1 - u^2 - v^2$$

$$r(1 - 2u + u^2) + rv^2 = 1 - u^2 - v^2$$

$$r - 2ru + ru^2 + rv^2 = 1 - u^2 - v^2$$

$$u^2(1 + r) - 2ru + v^2(1 + r) = (1 - r)u^2$$

$$-2ru/(1 + r) + v^2 = (1 - r)/(1 + r)$$

$$[u - r/(1 + r)]^2 + v^2 = (1 - r)/(1 + r) + r^2/(1 + r)^2$$

last term on RHS added to balance added term on LHS

Hence

$$[u - r/(1 + r)]^2 + v^2 = [(1 - r)(1 + r) + r^2]/(1 + r)^2 = [1 - r^2 + r^2]/(1 + r)^2 = 1/(1 + r)^2$$

We now have rearranged **eqn 1** into a standard circle format :-

$[u - r/(1 + r)]^2 + v^2 = 1/(1 + r)^2$

ie circle centred at  $(r/(1 + r), 0)$  with radius  $1/(1 + r)$

Thus circles can be drawn for various values of r with all centred on the u axis

Consider now **eqn 2** and rearranging it into that of the eqn for a circle :-

$$x = 2v/[(1 - u)^2 + v^2]$$

Hence

$$x[(1 - u)^2 + v^2] = 2v$$

$$x[1 - 2u + u^2 + v^2] = 2v$$

$$[1 - 2u + u^2 + v^2] = 2v/x$$

$$u^2 - 2u + v^2 - 2v/x = -1$$

$$(u - 1)^2 + (v - 1/x)^2 = -1 + 1 + 1/x^2 = 1/x^2$$

2 terms on RHS added to balance added term on LHS

We have now rearranged eqn 2 into a standard circle format :-

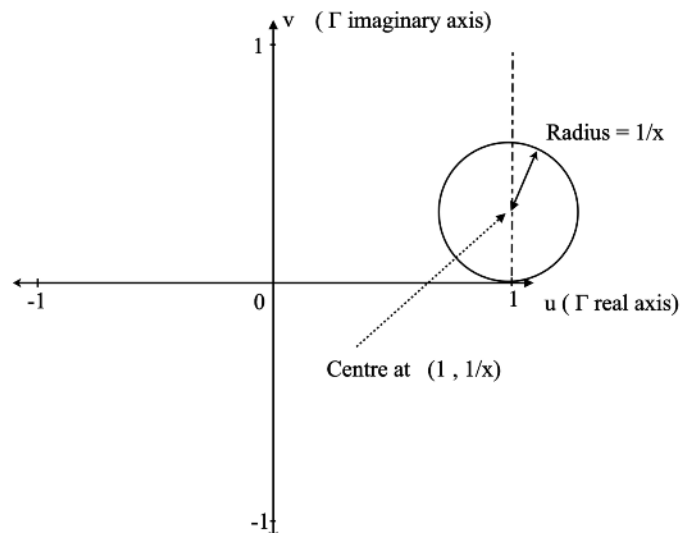
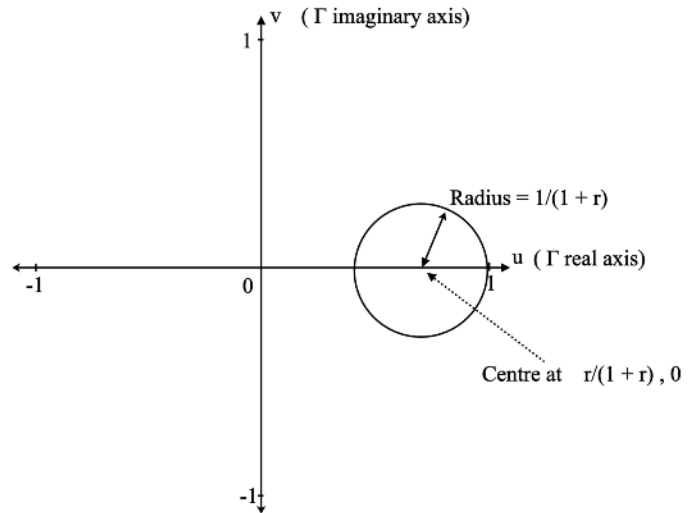
$$(u - 1)^2 + (v - 1/x)^2 = 1/x^2$$

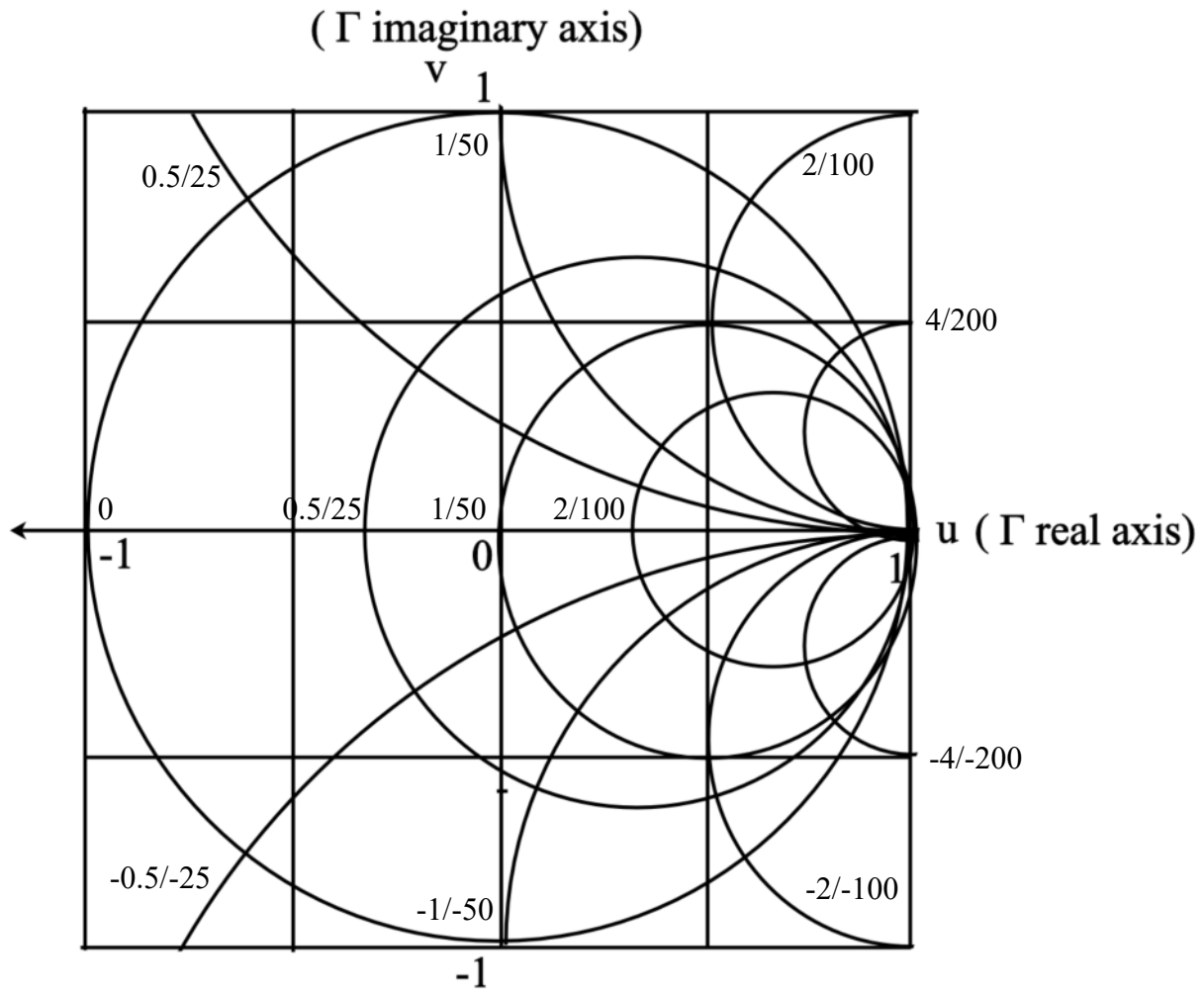
ie circle centre at  $(1, 1/x)$  with radius  $1/x$

Thus circles can be drawn for various values of x all centred on the u = 1

Finally, circles for various values of both r and x (namely 0.5, 1, 2, and 4 equivalent to 25, 50, 100 and 200 ohms) are plotted using the u and v Reflection Coefficient  $\Gamma$  component axes as shown on the following page.

Each circle is labelled with its normalised/actual value( ohms)

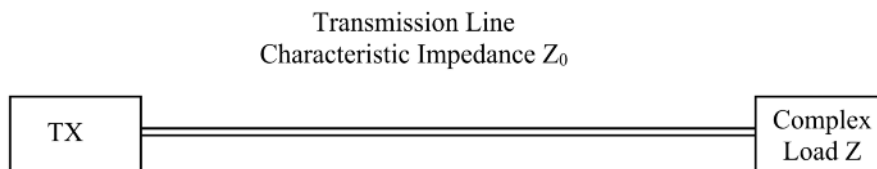




Will now consider how this Smith Chart can be used in a typical situation

**EXAMPLE OF USEFULNESS OF THE SMITH CHART**

Consider an antenna fed via a known length of eg RG8 coaxial cable having a Characteristic Impedance  $Z_0$  of 50 ohms. Assume that the antenna is required to operate on eg 14.2MHz but measurement of SWR back in the shack shows an SWR of 2.6:1 indicating that the antenna is a mismatch to the 50 ohms coax. This could be that although the antenna is resonant but its resistance differs from 50 ohms. However, it is more likely that the antenna is not resonant and presents both resistive and reactive impedances to the coax. This could be due to the antenna being the wrong length or that it is detuned by nearby objects.

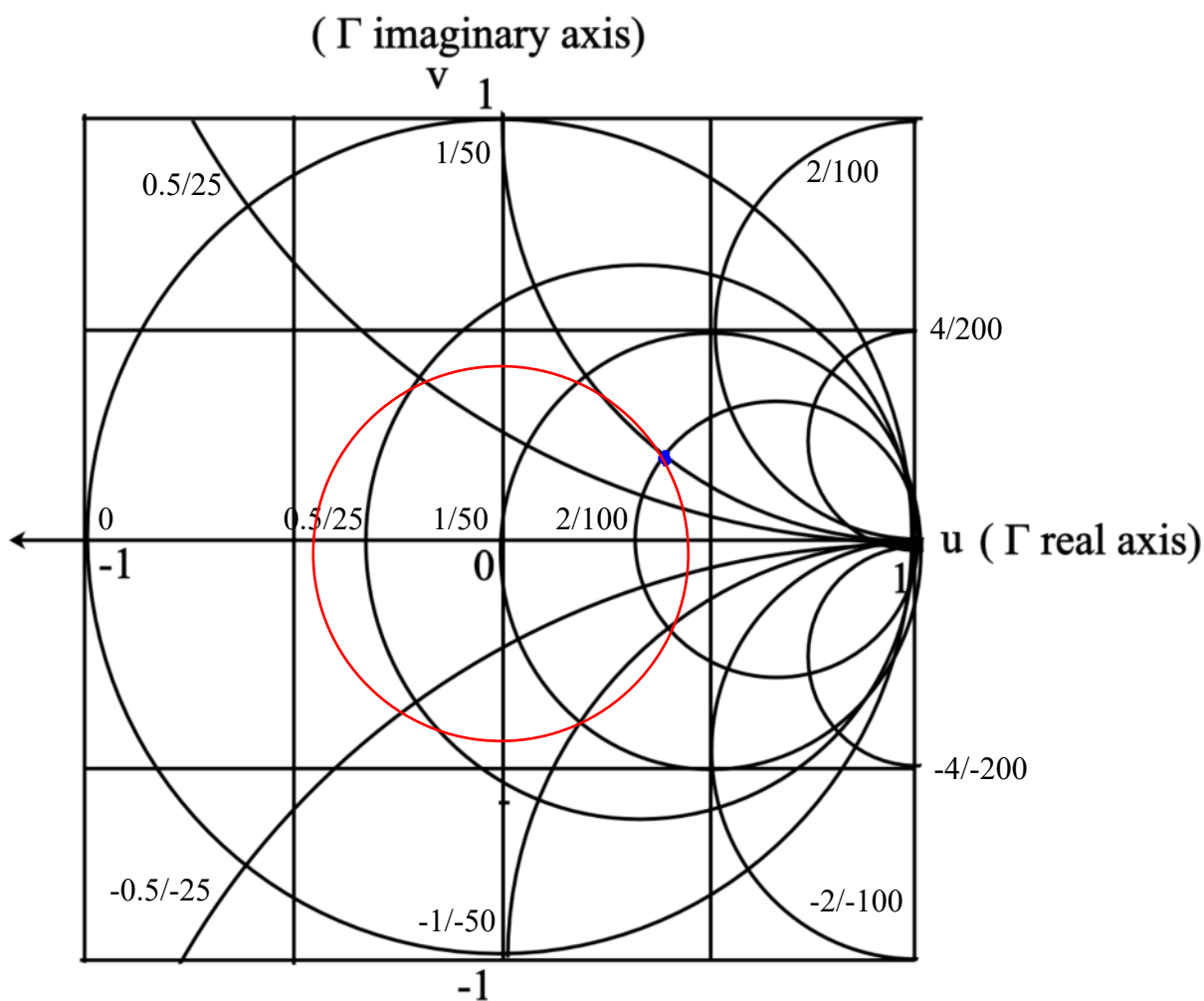


To analyse the problem, a VNA eg the very reasonably priced NanoVNA, was connected to the shack end of the coax and to gave measured values of resistance  $R = 100$  ohms and reactance  $X = +50$  ohms.

It was stated above that the length of RG8 was known which was physically 35m. However, length is required in terms of wavelength at 14.2MHz which in free space is  $300/14.2 = 21.127$ m. Since the speed of a radio wave in RG8 coax is slower than in free space (0.8 times slower according to its spec) then the wavelength in RG8 is in fact  $(0.8 \times 300)/14.2 = 16.9$ m.

Hence the 35m physical length of RG8 equates to  $35/16.9 = 2.07$  electrical wavelengths.

Consider now a circle drawn on the Smith Chart centred on the Chart origin at its centre and passing through the point where the measured resistance R value (100 ohms) circle crosses the measured value X (+50 ohms) circle (positive value indicates inductive implying that the antenna is too long) as shown below :-



The red circle obviously has a constant **radius =  $\sqrt{u^2 + v^2}$**  where u and v are values of any point on the circle. From the above Smith Chart the radius is approx 0.45.

Since the Reflection Coefficient  $\Gamma = u + jv$  then the magnitude of  $\Gamma$  (referred to as k at the beginning of this article) is given by  **$k = \sqrt{u^2 + v^2}$**  and hence k (magnitude of the Reflection Coefficient) = the radius of the red circle ie k = approx 0.45.

We previously found that  $SWR = (1 + k)/(1 - k)$  and hence  $SWR = (1 + 0.45)/(1 - 0.45) = 1.45/0.55 = 2.6$  as measured in the shack.

Since the red circle has a constant radius, then it follows that at any point around this circle, the Reflection Coefficient k is also constant and equal to 2.6 as therefore is also the SWR at 2.6.

For a **loss free** transmission line, the Reflection Coefficient and SWR remain constant everywhere along the line.

Hence the red circle plots values of r and x everywhere along the transmission line with the plotted blue point representing the transmitter end of the line. We also know that the impedance of the line is repeated every electrical half wavelength and hence one complete revolution around the circle equates to a distance of half an electrical wavelength. Travelling in an anticlockwise direction around the circle equates to travelling along the line in the direction of the load (antenna).

Therefore to find the impedance at the antenna end of 2.07 electrical wavelengths of RG8 we simply need to rotate around the circle a total of 4 times plus a tiny bit (insignificant!) and hence the impedance at the antenna end of the RG8 cable is virtually the same as that measured in the shack (**ASSUMING A LOSS FREE LINE**)